Assignment of Chapter 5

1. Given a rate 2/3 convolutional code, the memory order is $m = 1$. The encoder consists of two shift registers, each with $m = 1$ delay element, along with three mod-2 adders. Let $u_t^{(i)}$ denote the i-th input of the encoder in the t-th time instant, where $i \in \{1, 2\}$. Thus, the message vector can be defined as

$$\underline{u} = \left(u_0^{(1)}, u_0^{(2)}, u_1^{(1)}, u_1^{(2)}, u_2^{(1)}, u_2^{(2)}, \dots\right),$$

or as the two input vectors

$$\underline{u}^{(1)} = \left(u_0^{(1)}, u_1^{(1)}, u_2^{(1)}, \dots\right),$$

and

$$\underline{u}^{(2)} = \left(u_0^{(2)}, u_1^{(2)}, u_2^{(2)}, \dots\right).$$

There are three generator sequences corresponding to each message vector. Let $c_t^{(j)}$ denote the j-th output of the encoder in the t-th time instant, where $j \in \{1, 2, 3\}$. Let us further define the generator sequence corresponding to the input $u_t^{(i)}$ and the output $c_t^{(j)}$ as $g_i^{(j)}$. The transfer functions are as follows:

$$\begin{cases} g_1^{(0)}(x) = 1 + x; \\ g_1^{(1)}(x) = x; \\ g_1^{(2)}(x) = 1 + x, \end{cases} \quad \text{and} \quad \begin{cases} g_2^{(0)}(x) = x; \\ g_2^{(1)}(x) = 1; \\ g_2^{(2)}(x) = 1. \end{cases}$$

   (a) Please determine its shift register encoder structure, state table and trellis.

   (b) Please determine the codeword of a message vector

$$\underline{u} = (1, 0, 1, 1, 0, 0, 1, 0).$$

   Solution tip: The codeword should follow the order $(c_t^{(1)}, c_t^{(2)}, c_t^{(3)})$, and so on.

2. Given a 4-state $(7, 5)_8$ convolutional code, the received word is

$$\underline{r} = (1, 1, 0, 1, 1, 1, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0).$$

   Please apply viterbi decoding algorithm to determine the estimation of message vector $\underline{u}$.

   Solution tip: The decoding computation can be performed using the Viterbi trellis, as shown in Figure 1.

3. Given a 2/3 TCM code whose encoder structure and mapping scheme are shown in Figure 2.

   (a) Please determine the trellis of the TCM code.

(b) Please determine the free distance of the TCM code.

(c) Please determine the codeword of a message vector

$$\underline{u} = (0, 1, 1, 0, 1, 1, 0, 1).$$

Note that codeword shall be a 8PSK symbol sequence.

(d) Please determine the asymptotic coding gain (in dB) that the TCM coded system is able to achieve over an uncoded QPSK system.

Solution tip: Each pair of bits is in the order (a_2, a_1), and tailing bits are in the form $(a_2, a_1) = (0, 0)$. The codeword should follow the order (c_3, c_2, c_1), and so on.

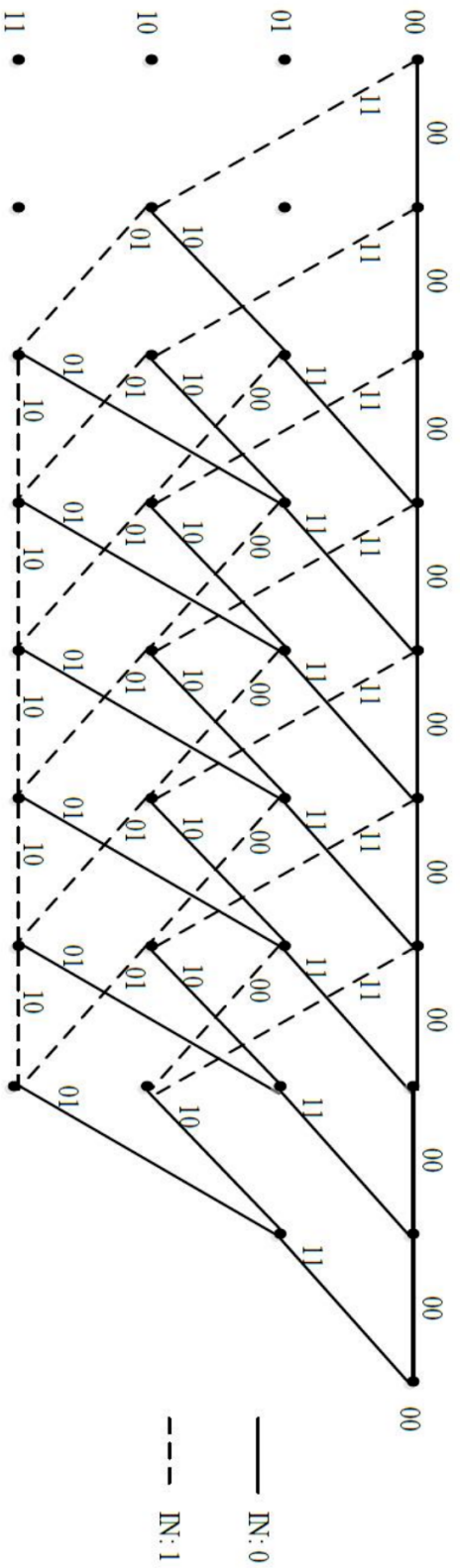


Figure 1: Viterbi trellis of the $(7, 5)_8$ convolutional code.

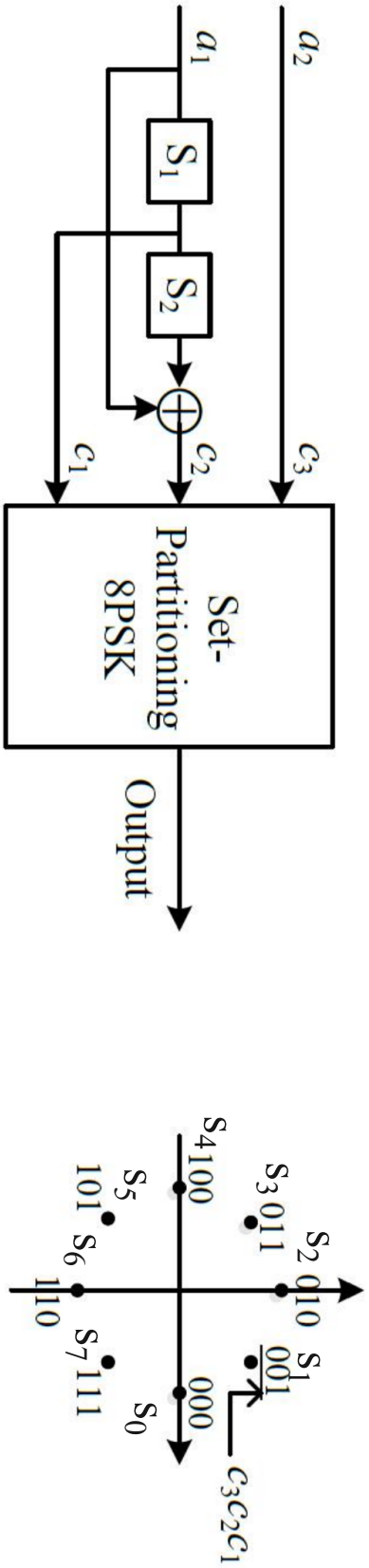


Figure 2: Rate 2/3 TCM encoder and the 8-PSK mapping scheme.